List of exercises for the laboratory 1

1. Show that set of vectors (1,0,1,0), (1,1,0,0), (0,1,1,1) (0,0,1,1) is a basis for R^4

2. Present a vector (2,0,-1,-2) as a linear combination of the vectors from the basis mentioned in p.1

3. Show, that if a classical "0" bit is represented by a vector (1,0) and a classical "1" bit is represented as a vector (0,1), that NOT operator is represented by the matrix

   $X = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

   Comment: that representation is further extended - the state of a quantum bit is a unit (length = 1) vector that is a linear combination of above mentioned vectors; the operations on quantum bits are represented by 2x2 unitary matrices. An unitary matrix does not change the length of a vector multiplied by it (see p.10).

4. Show that three Pauli matrices ($\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$) and the unit operator $\mathbf{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are linearly independent in linear space of matrices 2x2 both for R^4 and C^4

5. Let $A = \begin{bmatrix} 1+i & 2+2i & 3+3i \\ 4+4i & 5+5i & 6+6i \\ 7+7i & 8+8i & 9+9i \end{bmatrix}$

   Calculate hermitian conjugate $A^\dagger = (A^*)^T$

6. Show that Pauli matrices are hermitian ($A = A^\dagger$).

7. Reminder. Calculate scalar product for vectors in R^3: $\vec{a} = (1, 2, 3), \vec{b} = (3, 1, 4)$

8. Vectors of complex coordinates. Dirac notation. Let $\psi = (1 - i, 2 - 2i)$ and $\phi = (3 + 3i, 4 + 4i)$

   - show column (ket) representation of $|\psi\rangle$
   - show row representation (dual or bra vector) of $\langle\psi|$
   - calculate scalar (i.e. inner or dot) product $\langle\psi|\phi\rangle$
   - calculate scalar (i.e. inner or dot) product $\langle\psi|\psi\rangle$
   - calculate norm (lenght) of ψ defined as $\sqrt[2]{\langle\psi|\psi\rangle}$
   - show martix representation of the operator: $|\psi\rangle\langle\phi|$

9. Show using matrix of complex numbers that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\psi = (\alpha, \beta)$, that $(A|\psi\rangle)^\dagger = \langle\psi|A^\dagger$

10. Show that matrix that satisfies condition $AA^\dagger = A^\dagger A = 1$ does not change the norm (lenght) of a vector. Use result from p.9

    Comment: Matrix that satisfies condition $AA^\dagger = A^\dagger A = 1$ is called unitary. An arbitrary unitary matrix 2x2 can be used as operation on qbit (also called 1-qbit gate).

11. Show that Pauli matrices are unitary.

12. Tensor product of vectors. Let $\psi = (1+i, 2+2i)$ and $\phi = (3+3i, 4+4i)$. Calculate $|\psi\rangle \otimes |\phi\rangle$.

    Comment: In a classical computer the state of a register consisting of many bits is represented by a tensor product of vectors (1,0) and (0,1) representing states of the particular single bits (see p.3).

    All possible vectors (i.e. classical register states) obtained in this way form a basis for a state of a quantum register. A state of a quantum register is a linear combination of all possible classical register states. This is also called quantum parallelism (a quantum state describes all possible classical states).

13. Tensor product of matrices A and B is a matrix $C = A \otimes B$, such that $A|\psi\rangle \otimes B|\phi\rangle = C(|\psi\rangle \otimes |\phi\rangle)$. Use the above information for vectors $\psi = (\alpha, \beta), \phi = (\gamma, \delta)$ to find matrix representation of $X \otimes X$.

    Comment: Operations on quantum registers (containing n qbits) are represented by nxn unitary matrices (n-qbits gates). In the actual design of quantum algorithms the class of allowed unitary transformations is almost always restricted to ones that can be built up out of tensor products of 1-Qbit gates or 2-Qbit gates. This is because of the technical problems of making higher order quantum gates.